

POLICE OFFICER
PREPARATION MATERIALS

5

Math Reasoning
Ability to reason through mathematical problems

1. A basketball team has won 50 games of 75 played. The team still has 45 games to play. How many of the remaining games must the team win in order to win 60% of all games played during the season?
 - a. 20
 - b. 21
 - c. 22
 - d. 25
 - e. 30

2. A rectangle and a triangle have equal areas. The length of the rectangle is 12 inches, and its width is 8 inches. If the base of the triangle is 32 inches, what is the length, in inches, of the altitude drawn to the base?
 - a. 6
 - b. 8
 - c. 9
 - d. 12
 - e. 16

3. A school has 18 classes with 35 students in each class. In order to reduce class size to 30, how many new classes must be formed?
 - a. 2
 - b. 3
 - c. 5
 - d. 6
 - e. 8

4. A bell rings every 2 hours, a second bell rings every 3 hours, and a third bell rings every 4 hours. If all 3 bells ring at 9:00 AM., at what time will all 3 bells next ring?
 - a. noon
 - b. 6:00 p.m.
 - c. 9:00 p.m.
 - d. 10:00 p.m.
 - e. Not enough information is given

5. A family spends 20% of its monthly income on food, 23% on rent, and 42% on other expenses and saves the balance. If the family saves \$360 per month, what is its monthly income?
- \$2,000
 - \$2,200
 - \$2,400
 - \$2,500
 - \$28,800
6. How many 4-inch by 8-inch bricks are needed to build a walk 6 feet wide and 24 feet long?
- 54
 - 600
 - 648
 - 840
 - 1,000
7. A room is 24 feet long, 18 feet wide, and 9 feet high. How many square yards of wallpaper are needed to paper the four walls of the room?
- 72
 - 84
 - 96
 - 180
 - 756
8. 228 people have registered for a sightseeing tour of the Grand Canyon. If a tour director can take 35 people on each tour, how many tour directors will be needed to accommodate everyone?
- 5
 - 6
 - 7
 - 8
 - 9
9. Leonard is travelling from New York to Nevada. The drive is 2,700 miles. He drove 300 miles the first day, 225 miles the second day, 375 miles on the third day. Based on his average speed, how many more days will he need to get to Nevada?
- 3 days
 - 6 days
 - 9 days
 - 12 days
 - 15 days

Math Reasoning

1. 22

2. 6

Explanation of Answer:

The team has played 75 games and will play 45 more games.

$$75 + 45 = 120$$

$$60\% \text{ of } 120 = 0.6 \times 120 = 72$$

The team must win 72 games, and it has already won 50 games. Therefore, the team must win $72 - 50 = 22$ more games.

3. 3

Explanation of Answer:

The number of students in the school $18 \times 35 = 630$. If there are to be 30 students in a class, the number of classes needed is $630 / 30 = 21$.

Therefore, the number of new classes needed is $21 - 18 = 3$.

4. 9:00 p.m.

Explanation of Answer:

The easiest way to do this is by looking at the third bell that only rings every 4 hours. You can immediately find a solution by multiplying Bell 3 \times Bell 2 $= 4 \times 3 = 12$. Because 12 is also evenly divisible by 2, this is a solution, but you don't know if this is the best solution.

To verify it is the best alternative, you must look at each time the third bell rings and determine if it's evenly divisible by the other two. Therefore, you would look at 4, 8, and 12.

Because 12 is the only bell that is evenly divisible by Bells 1 and 2, you know the bells will all ring every 12 hours. 9:00 AM $= 12$ hours $= 9:00$ PM

5. \$2,400

Explanation of Answer:

$$\text{Expenditures: } 20\% + 23\% + 42\% = 85\%$$

$$\text{Savings: } 100\% - 85\% = 15\%$$

Let x = family's monthly income. Then $.15x = \$360$, so $x = \$360 / .15 = \$2,400$

6. 36

Explanation of Answer:

Width of walk is 6 ft., or $6 \times 12 = 72$ in. Width of each brick is 4 in. Number of bricks that can be fitted along the width is $72 / 4 = 18$.

Length of walk is 24 ft., or $24 \times 12 = 288$ in. Length of each brick is 8 in. Number of bricks that can be fitted along the length is $288 \div 8 = 36$

7. **84 sq yds**

Explanation of Answer:

Area of front wall =

$$9 \times 24 = 216 \text{ sq. ft.}$$

Area of back wall

$$9 \times 24 = 216 \text{ sq. ft.}$$

Area of side wall =

$$9 \times 18 = 162 \text{ sq. ft.}$$

Area of other side wall =

$$9 \times 18 = 162 \text{ sq. ft.}$$

Total area of walls =

$$216 + 216 + 162 + 162 = 756 \text{ sqft.}$$

$$756 / 9 = 84 \text{ sq. yd.}$$

8. **7**

Explanation of Answer:

$228 / 35 = 6.51$, so 7 tour directors will be needed.

9. **6 days**

Explanation of Answer:

Leonard is travelling at a speed of 300 miles per day. He has already travelled 900 miles leaving only 1,800 left to go. At this rate it will take him 6 more days of travelling.

Basic Math Skills

Our number system is a *decimal system*, so called because it is based on the number 10 (*decem* is Latin for ten). Any number can be written using the proper choice and combination of ten symbols, 0 through 9.

The decimal system is a *place value* system. This means that the value of each digit depends in part on its location, or *place*, in the number. As the diagram below shows, each place or position is labeled. As you move to the left in a decimal number, each place is worth ten times the previous place; as you move to the right, each place is worth one-tenth of the previous place. We can analyze the decimal number 5,183.604 as follows:

5	1	8	3	.	6	0	4
t	h	t	u	d	t	h	t
h	u	e	n	e	e	u	h
o	n	n	i	c	n	n	o
u	d	s	t	i	t	d	u
s	r		s	m	h	r	s
a	e			a	s	e	a
n	d			l		d	n
d	s					t	d
s				p		h	t
				o		s	h
				i			s
				n			
				t			

The number 5,183.604 can be fully expressed as five thousands, one hundred, eight tens, three units, six tenths, no hundredths, and four thousandths – or read in the more conventional way as five thousand, one hundred eighty-three, and six hundred four thousandths.

Rounding Decimal Numbers

Calculation can result in answers with more places than are useful. Since our money system, for example uses dollars and cents, an answer of \$31.276 is not usable without rounding. Dollar and cents are rounded to the nearest hundredth, at least. Other numbers can be rounded as desired.

To round a number, drop all the digits to the right of the place to which the number is to be rounded. If the first digit to be dropped is 5 or more, increase the last digit to be kept by one. If the first digit to be dropped is less than 5, drop the digit.

5.732 rounded to tenths is 5.7

5.752 rounded to tenths is 5.8

The sample problem below illustrates how a number can be rounded in several ways. Rounding to the nearest whole number is also expressed as rounding to the nearest unit or nearest integer.

Sample Problem:

Round the number 8,612.093 to the nearest (a) thousand; (b) hundred; (c) ten; (d) whole number; (e) tenth; (f) hundredth.

Solution:

- a. 9,000 b. 8,600 c. 8,610 d. 8,612 e. 8612.1 f. 8,612.09

Adding Whole Numbers and Decimals

Addition is a matter of technique. The goals of the successful "adder" are speed and accuracy. One useful speed-building technique is to look for combination of ten in a series to be added. For instance, let's add these numbers:

$$6 + 9 + 4 + 1 + 3 + 8 + 7$$

The slower process goes "6 + 9 = 15; 15 + 4 = 19, etc." The rapid adder says "6 + 4 = 10; 9 + 1 = 10; 3 + 7 = 10; 3 10's = 30 + 8 = 38." Other combinations may also become useful to you as you form your own speed building habits.

Alignment of numbers around the decimal point is also essential to successful addition. To accomplish this, (1) line up all decimal points in one column; (2) equalize the number of digits by filling in zeros for those numbers with fewer decimal digits than the longest number. In the sample problem below, 37.5 becomes 37.500 in order to align the number correctly.

Sample Problem:

Add 2,034.5, 571.835, .082, 37.5, and 7.94. Round the answer to the nearest hundredth.

Solution:

$$\begin{array}{r} 2,034.500 \\ 571.835 \\ .082 \\ 37.500 \\ 7.940 \\ \hline 2,651.857 = 2,651.86 \end{array}$$

The addition shown above is known as *vertical addition*, because we added down. Other problems call for adding across, or *horizontal addition*. Vertical and horizontal addition are often used in the same problem to provide a double check on accuracy.

Subtracting Whole Number and Decimals

Subtraction is the second of the basic arithmetic skills. With a problem such as the one shown below we find little difficulty.

$$\begin{array}{r} 597 \\ -315 \\ \hline 282 \end{array}$$

But to subtract 287 from 964, we must use the technique of borrowing from columns to the left. We can borrow because of place values. The "1" we borrow for 6 in step (a) below is really 10 which we add to the 4. We are breaking 64 into 50 and 14.

$$\begin{array}{r} 8 \quad 1_5 \quad 1 \\ 9 \quad 6 \quad 4 \\ -3 \quad 8 \quad 7 \\ \hline 5 \quad 7 \quad 7 \end{array} \quad \begin{array}{l} \text{(a) Borrow 1 from the 6, make it 5, and change 4 to 14 (add 10).} \\ 14 - 7 = 7. \end{array}$$

$$\begin{array}{r} 9 \quad 6 \quad 4 \\ -3 \quad 8 \quad 7 \\ \hline 5 \quad 7 \quad 7 \end{array} \quad \begin{array}{l} \text{(b) Borrow 1 from the 9, make it 8, and change the 5 to 15} \\ \text{(add 10). } 15 - 8 = 7. \end{array}$$

(c) $8 - 3 = 5$

In decimal subtraction, as in addition, be sure to align the numbers around the decimal point. Then, equalize the number of digits in each number by filling in zeroes where necessary. To subtract 136.0472 from 3,148.09, change the latter number to 3,148.0900 in order to equalize the digits. If the answer is required to be in hundredths, round to two decimal places.

$$\begin{array}{r} 3,148.0900 \\ - 136.0472 \\ \hline 3,012.0428 = 3,012.04 \end{array}$$

Subtraction is often carried out horizontally in business.

Multiplying Whole Numbers and Decimals

Multiplication is the most widely used of the four basic skills in business mathematics. A knowledge of shortcut techniques in multiplication can improve your speed and accuracy.

To multiply by 10 or a "power" of 10 (100, 1,000, and so on), move the decimal point of the number to be multiplied (the *multiplicand*) to the right as many places as there are zeros in the *multiplier*. If the multiplicand is a whole number, simply add zeroes to it. For example:

$$\begin{array}{l} 83.75 \times 10 = 837.5 \\ 68 \times 100 = 6,800 \end{array}$$

To multiply by a multiple of 10 (or 100, etc.), first deal with the zeros. For example:

$$509 \times 30 = 509 \times 10 \times 3 = 5,090 \times 3 = 15,270$$

This procedure helps by increasing the number of problems we can do mentally. Other problems do require a written procedure. To multiply 864 by 600 most efficiently:

$$\begin{array}{r} 864 \\ \times 600 \\ \hline 518,400 \end{array}$$

(a) Set zeros to the right
(b) Copy zeros on answer line
(c) Multiply $6 \times 864 = 5,184$

Be exact in your handling of decimal points when multiplying. The number of places in the answer is the total of the number of places in both the multiplicand and the multiplier. When multiplying 7.63 by .4719, the answer will have $2 + 4 = 6$ places to the right of the decimal point, as shown below.

$$\begin{array}{r} .4719 \quad (4 \text{ places}) \\ \times 7.63 \quad (2 \text{ places}) \\ \hline 14157 \\ 28314 \\ \hline 33033 \\ 3600597 = 3.600597 \quad (6 \text{ places}) \end{array}$$

Dividing Whole Numbers and Decimals

Long division problems are relatively rare in business, but when they do occur, you should know how to solve them.

Let's look at the division of 603 by 32. The *divisor* is 32; the *dividend*, 603.

(a) 32 goes into 60 one time.

(b)	1 X 32 = 32;	→	<u>18</u>		
	60 - 32 = 28;	→	32 603		
	Bring down the		<u>32</u> ↓		
	3.		283 ←	(c)	32 goes into 283 8 times.
			<u>256</u> ←	(d)	32 X 8 = 256; the difference
			27		is 27.

The answer is 18, with a remainder of 27. If the problem called for rounding to the nearest unit, since over half of 32 remains (27/32), 19 would be the final answer. Or, division could be carried one more place and then rounded back to the nearest unit.

If the problem above had called for rounding to tenths, you would have carried the division two places further in order to round back to tenths.

$$\begin{array}{r}
 \underline{18.84} \\
 32 \overline{)603.00} \\
 \underline{32} \\
 283 \\
 \underline{256} \\
 270 \\
 \underline{256} \\
 140 \\
 \underline{128} \\
 12
 \end{array}$$

Rounding 18.84 to tenths gives us an answer of 18.8. To divide by a divisor with numbers to the right of the decimal point, follow these preliminary steps:

1. Move the decimal point of the divisor all the way to the right in order to make it a whole number.
2. Move the decimal point of the dividend to the right the same number of places that you moved the decimal point of the divisor in step 1. Add zeros if necessary.
3. Place the decimal point of the answer above the decimal point in the new dividend.

FRACTIONS

A fraction consists of two (2) whole numbers separated by a bar and represents a part of a whole, a division, or a ratio. For example 5/8 may mean:

- 5 of 8 parts of a whole remain, 3 of the 8 having been used;
- 5 divided by 8;
- 5 shares in a pool are to be divided among 8 people – the ratio of shares to people is 5 to 8.

The fraction bar is either horizontal (-) or slanted (/). If the bar is horizontal, the number above it is called the *numerator* and the number below it is called the *denominator*. If the bar is slanted, the number to the left of it is the numerator and the number to the right of it is the denominator. The numerator and the denominator are the *terms* of the fraction.

Fractions can be expressed in an infinite number of ways, so long as the terms are multiplied or divided by the same *factor* (whole number).

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8} = \frac{6 \times 3}{8 \times 3} = \frac{18}{24} = \frac{18 \div 2}{24 \div 2} = \frac{9}{12}$$

Reducing to Lowest Terms

We say that a fraction has been reduced to the lowest terms when there is no factor in common by which the numerator and denominator can be divided evenly.

To reduce a fraction to the lowest terms, divide both the numerator and denominator by common factors until further even division is impossible. For example, to reduce 750/975 to lowest terms:

- a. Divide by the obvious factor, 5.

$$\frac{750}{975} = \frac{750 \div 5}{975 \div 5} = \frac{150}{195}$$

- b. Divide by 5 again.

$$\frac{150}{195} = \frac{150 \div 5}{195 \div 5} = \frac{30}{39}$$

- c. The numerator (30) and the denominator (39) can both be divided evenly by 3.

$$\frac{30}{39} = \frac{30 \div 3}{39 \div 3} = \frac{10}{13}$$

The numbers 10 and 13 have no common factor, so the fraction 750/975 has been reduced to the lowest terms.

Adding Fractions

All four basic arithmetic processes used with whole numbers and decimal numbers can also be used with fractions. To add two (2) fractions with the *same denominator*, simply add the numerators while keeping the common denominator.

Example: $\frac{1}{8} + \frac{5}{8} = \frac{6}{8}$

Since 6 and 8 can both be divided evenly by 2, the fraction 6/8 should then be reduced to the lowest terms of 3/4.

To add two (2) fractions that have *different denominators*, one or both fractions must be changed to get the same denominator for both. The new denominator should be the *lowest common denominator* in order to minimize later reductions. The lowest common denominator (l.c.d.) is the smallest number into which both of the original denominators can be evenly divided.

In adding $\frac{1}{4}$ and $\frac{1}{8}$, one of the original denominators can be divided by the other: the 8 by the 4. Only the 4 needs to be re-expressed, then, as an 8.

$$\frac{1}{4} + \frac{1}{8} = \frac{1 \times 2}{4 \times 2} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8} = \frac{3}{8}$$

In adding $\frac{3}{5}$ and $\frac{2}{7}$, neither denominator can be divided by the other. Both fractions must be converted into equivalent fractions with a common denominator. What numbers can be divided by both 5 and 7? One is 70. Is this the *lowest* common denominator? A continuation of this reasoning will eventually lead to the selection of 35 as the lowest. Both fractions will then be converted to 35ths.

$$\frac{3}{5} + \frac{2}{7} = \frac{3 \times 7}{5 \times 7} + \frac{2 \times 5}{7 \times 5} = \frac{21}{35} + \frac{10}{35} = \frac{31}{35}$$

The fraction $\frac{31}{35}$ is at its lowest terms because it cannot be reduced evenly.

Subtracting Fractions

Fractions must have the same denominator to be subtracted. Once the denominators are identical, the difference between the numerators is found and placed over the common denominator.

To subtract $\frac{4}{9}$ from $\frac{7}{9}$:

$$\frac{7}{9} - \frac{4}{9} = \frac{3}{9}, \text{ reduced to } \frac{1}{3}$$

To subtract $\frac{3}{5}$ from $\frac{5}{8}$:

$$\frac{5}{8} - \frac{3}{5} = \frac{5 \times 5}{8 \times 5} - \frac{3 \times 8}{5 \times 8} = \frac{25}{40} - \frac{24}{40} = \frac{1}{40}$$

Multiplying Fractions

To multiply fractions, multiply like terms. Multiply numerator by numerator, and denominator by denominator. Reduce all answers to lowest terms. To multiply $\frac{5}{7}$ by $\frac{2}{3}$:

$$\frac{5}{7} \times \frac{2}{3} = \frac{5 \times 2}{7 \times 3} = \frac{10}{21}$$

To multiply a whole number by a fraction, change the whole number to a fraction by placing the number over 1. The number 6 as a fraction = $\frac{6}{1}$. To multiply 6 by $\frac{1}{9}$:

$$6 \times \frac{1}{9} = \frac{6}{1} \times \frac{1}{9} = \frac{6 \times 1}{1 \times 9} = \frac{6}{9} = \frac{2}{3}$$

Dividing Fractions

To divide one (1) fraction by another, invert the second fraction and multiply. To divide $\frac{3}{5}$ by $\frac{2}{3}$:

$$\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2} = \frac{3 \times 3}{5 \times 2} = \frac{9}{10}$$

To divide a whole number by a fraction, change the whole number to a fraction, invert the second fraction, and multiply.

$$9 \div \frac{3}{4} = \frac{9}{1} \div \frac{3}{4} = \frac{9}{1} \times \frac{4}{3} = \frac{36}{3} = \frac{12}{1} = 12$$

Mixed Numbers and Improper Fractions

A *mixed number* is a number that contains both a whole number and a fraction; therefore, $7 \frac{1}{3}$ is a mixed number. To work with a mixed number, it is often most useful to change it to fraction form. The resulting fraction will have a numerator larger than its denominator and is called an *improper fraction*.

To convert $7 \frac{1}{3}$ to an improper fraction, treat the problem as the addition of two (2) fractions ($7/1 + 1/3$) with different denominators.

$$\frac{7}{1} + \frac{1}{3} = \frac{7 \times 3}{1 \times 3} + \frac{1}{3} = \frac{21}{3} + \frac{1}{3} = \frac{22}{3}$$

A shorter method is to realize that the number 7, expressed in thirds, contains 7×3 thirds or 21 thirds. Then,

$$(1) \quad 7 \times 3 = 21 \text{ thirds}$$

$$(2) \quad 21/3 + 1/3 = 22/3$$

To reduce an improper fraction, divide the numerator by the denominator. If the division is even, the answer is a whole number; if uneven, the result is a mixed number.

$$132/6 = 22$$

$$133/6 = 22 \frac{1}{6}$$

Cancellation in Multiplications and Division

Multiple 730 by $4 \frac{1}{5}$. If you follow the rules to this point.

$$730 \times 4 \frac{1}{5} = \frac{730}{1} \times \frac{21}{5} = \frac{15,330}{5} = 3,066$$

The procedure is correct, but working with such large numbers increases the likelihood of marking errors. Consider a simpler example: $\frac{3}{4} \times \frac{8}{3}$.

$$\frac{3}{4} \times \frac{8}{3} = \frac{3 \times 8}{4 \times 3}$$

We could continue by multiplying 3×8 and 4×3 . But by looking at the numbers closely, we can save time and minimize errors. Both numbers have common factors that can be equally reduced or canceled. Since 3 is a factor of both, divide both 3's by 3 obtaining 1.

$$\begin{array}{r} 1 \\ 3 \times 8 \\ \hline 4 \times 3 \\ 1 \end{array}$$

Four (4) is a factor of 4 and 8. Divide 4 and 8 by 4. Then multiply the reduced numbers.

$$\begin{array}{r} 1 \quad 2 = \quad = 2 \\ \hline 3 \text{ X } 8 \quad 2 \\ 4 \text{ X } 3 \quad 1 \\ \hline 1 \quad 1 \end{array}$$

Returning to $730 \times 4 \frac{1}{5}$:

$$\begin{array}{r} 146 \text{ X } 21 = 3,066 \\ \hline 730 \text{ X } 21 \\ \hline 1 \quad 5 \\ \quad 1 \end{array}$$

The figures are still large, but dividing 730 by 5 is less likely to involve errors than dividing 15,330 by 5.

Ratios

A ratio is a comparison of one number to another. A 2 to 1 ratio, written as either 2/1 or 2:1, means that the first number is twice as large as the second. If the second number is 200, the first is $2 \times 200 = 400$. The ratio of the second to the first is 1:2 or $\frac{1}{2}$.

The most common use of ratios in business mathematics is for comparison of a part to a whole or one part to another. For example, if a firm has 300 employees, 100 of whom are female and 200 of who are male:

<u>THE RATIO OF</u>	<u>IS</u>
employees to females	300/100 or 3:1
employees to males	300/200 or 3:2 or 1.5 to 1
males to employees	200/300 or 2:3 or 2/3
females to employees	100/300 or 1:3 or 1/3
males to females	200/100 or 2:1
females to males	100/200 or 1:2 or .5:1

Consider a variation of this problem. If a firm has 600 workers, with a 5:1 ratio of males to females, how many of each sex are employed?

A 5:1 ratio means five "part" male to every one "part" female. There are altogether six parts, five of which ($\frac{5}{6}$) are male and one of which ($\frac{1}{6}$) is female. Thus, $\frac{5}{6} \times 600 = 500$ males and $\frac{1}{6} \times 600 = 100$ females.

Sample Problem

A firm has 300 sales workers and 600 production workers.

- a. What fraction of the staff is in sales?
- b. What is the ratio of production to sales workers?

Solution

- a. Total work force = $300 + 600 = 900$
Sales/work force = $300/900 = 1/3$
- b. Production: sales = $600/300 = 2:1$

The sample problem below shows how to use fractions in a business problem.

A process to produce 600 pounds of Gallion requires a combination of 3 metals: $1/3$ Rosine, $1/8$ Lomelite, and the rest Anderite. Find the number of pounds of each metal used.

Rosine:
$$\frac{1}{3} \times \frac{600}{1} = 200 \text{ pounds}$$

Lomelite:
$$\frac{1}{8} \times \frac{600}{1} = 75 \text{ pounds}$$

Anderite:
$$\frac{1}{3} + \frac{1}{8} = \frac{11}{24}$$

$$1 = \frac{24}{24} - \frac{11}{24} = \frac{13}{24}$$

$$\frac{13}{24} \times \frac{600}{1} = 325 \text{ pounds}$$

Check: $200 + 75 + 325 = 600$

- Notes: (1) Each calculation of pounds was based on 600 pounds. The number 600 was expressed as the fraction $600/1$ and cancellation was used.
(2) To find the answer for Anderite, the first two (2) answers were not used. Had either answer been wrong, the Anderite answer would have been wrong.
(3) The total of the fractions – 1 – was expressed as $24/24$ in order to subtract $11/24$.
(4) A final check was included to assure accuracy.

CONVERTING FRACTIONS, DECIMALS AND PERCENTS

Part of a whole can be expressed as a fraction, a decimal, or a percent. The ability to convert from one form to another is often required in business problems. Each type of conversion is explained in this section.

Converting Fractions to Decimals

To convert a fraction to a decimal number, divide the numerator by the denominator. Carry out the division as far as required, inserting a decimal point and adding zeroes when necessary. To convert $5/8$ to decimal form, rounding to thousandths, divide 5.0000 by 8. Divide by the denominator.

$$\begin{array}{r} .6250 \\ 8 \overline{) 5.0000} \end{array}$$

Rounded to thousandths, the answer is .625. Another way of expressing this answer is $.62\frac{1}{2}$.

Converting Decimals to Percents

This type of conversion is a simple procedure: move the decimal point of the decimal number 2 places to the right, and add a percent sign.

$$.763 = 76.3\%$$

$$2.05 = 205\%$$

$$.06 = 6\%$$

$$.003 = .3\%$$

$$9 = 900\%$$

Converting Fractions to Percents

To convert a fraction to a percent, first convert the fraction to decimal form, and then convert the decimal number to a percent.

$$\frac{3}{7} = \frac{.42}{3.00} = \frac{.42}{7} = \frac{42}{7} = 42 \frac{6}{7} \%$$

Converting Decimals to Fractions

A decimal number is actually a fraction. The number .6 when read aloud is read as "six tenths" and can be expressed as $\frac{6}{10}$; .05 can be expressed as $\frac{5}{100}$. The conversion from decimal to fraction form relies on this alternate expression. Either write the number as a fraction and reduce to lowest terms.

$$.008 = \frac{8}{1000} = \frac{1}{125}$$

Or follow these three steps: (1) omit the decimal point; (2) place the number over 1 followed by as many zeros as there are digits following the decimal point; (3) reduce to the lowest terms.

$$.25 = \frac{25}{1 + 2 \text{ zeros}} = \frac{25}{100} = \frac{1}{4}$$

$$.625 = \frac{625}{1 + 3 \text{ zeros}} = \frac{625}{1000} = \frac{5}{8}$$

Converting Percents to Decimals

To convert a percent to decimal form, reverse the procedure used in converting a decimal to a percent. First, drop the percent sign; then move the decimal point 2 places to the left.

$$75\% = .75$$

$$62.5\% = .625$$

$$.7\% = .007$$

$$48 \frac{1}{3}\% = .48 \frac{1}{3}$$

$$108\% = 1.08$$

Converting Percents to Fractions

This is another two-step conversation procedure. First, convert the percent to decimal form; then convert the decimal to a fraction.

$$30\% = .30 = \frac{30}{100} = \frac{3}{10}$$

To convert a percent containing a fraction (such as 57 1/7%) to a fraction, an additional step is needed. First,

$$57 \frac{1}{7} \% = \frac{57 \frac{1}{7}}{100} = \frac{57 \frac{1}{7} \times 7}{100 \times 7} = \frac{57 \frac{1}{7} \times 7}{700}$$

Next, multiply both terms of this fraction by the denominator of the fraction in the mixed term, that is, 7. Finally, reduce to lowest terms.

$$\frac{57 \frac{1}{7} \times 7}{100 \times 7} = \frac{57 \frac{1}{7} \times 7}{700} = \frac{57 \frac{1}{7} \times 7}{700} = \frac{400}{700} = \frac{4}{7}$$

Refer to Table 1 below for help in the conversion of the more common fractions, decimals, and percents. For example, let's assume that you want to find the percent equivalent of 6/11. Look at the column headed 6 (as the numerator) and the row headed 11 (as the denominator). The intersection of the column and the row show 54 6/11% as the answer.

		Table 1 FRACTION, DECIMAL, AND PERCENT EQUIVALENTS										
		Numerator										
		1	2	3	4	5	6	7	8	9	10	11
D e n o m i n a t o r	2	.50 50%										
	3	.33 1/3 33 1/3%	.66 2/3 66 2/3%									
	4	.25 25%	.50 50%	.75 75%								
	5	.20 20%	.40 40%	.60 60%	.80 80%							
	6	.16 2/3 16 2/3%	.33 1/3 33 1/3%	.50 50%	.66 2/3 66 2/3%	.83 1/3 83 1/3%						
	7	.14 2/7 14 2/7%	.28 4/7 28 4/7%	.42 6/7 42 6/7%	.57 1/7 57 1/7%	.71 3/7 71 3/7%	.85 5/7 85 5/7%					
	8	.12 1/2 12 1/2%	.25 25%	.37 1/2 37 1/2%	.50 50%	.62 1/2 62 1/2%	.75 75%	.87 1/2 87 1/2%				
	9	.11 1/9 11 1/9%	.22 2/9 22 2/9%	.33 1/3 33 1/3%	.44 4/9 44 4/9%	.55 5/9 55 5/9%	.66 2/3 66 2/3%	.77 7/9 77 7/9%	.88 8/9 88 8/9%			
	10	.10 10%	.20 20%	.30 30%	.40 40%	.50 50%	.60 60%	.70 70%	.80 80%	.90 90%		
	11	.09 1/11 9 1/11%	.18 2/11 18 2/11%	.27 3/11 27 3/11%	.36 4/11 36 4/11%	.45 5/11 45 5/11%	.54 6/11 54 6/11%	.63 7/11 63 7/11%	.72 8/11 72 8/11%	.81 9/11 81 9/11%	.90 10/11 90 10/11%	
	12	.08 1/3 8 1/3%	.16 2/3 16 2/3%	.25 25%	.33 1/3 33 1/3%	.41 2/3 41 2/3%	.50 50%	.58 1/3 58 1/3%	.66 2/3 66 2/3%	.75 75%	.83 1/3 83 1/3%	.91 2/3 91 2/3%

Practice Math Problems

1. Divide $1 \frac{2}{3}$ by $1 \frac{1}{9}$:

2. Divide $12 \frac{1}{4}$ by $15 \frac{3}{4}$:

3. Solve the following addition problems:

a. $\begin{array}{r} 326.93 \\ 25.001 \\ \hline 1,711.9 \end{array}$	b. $\begin{array}{r} 25.39 \\ 17.009 \\ \hline 0.004 \end{array}$	c. $\begin{array}{r} 67 \\ 87 \\ \hline 4,000.00 \end{array}$	d. $\begin{array}{r} 190.95 \\ 305.5123 \\ \hline 777.6 \end{array}$
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e. $\begin{array}{r} \$6,000.00 \\ \$6,500.00 \\ \$7,000.00 \\ \hline \$8,500.00 \end{array}$	f. $\begin{array}{r} \$18,000.00 \\ \$36.13 \\ \$22.00 \\ \hline \$37.48 \end{array}$	g. $\begin{array}{r} 27,147.901 \\ 0.000876 \\ 16 \\ \hline 48.1955 \end{array}$	h. $\begin{array}{r} 60.108 \\ 910 \\ 1018 \\ \hline 12 \end{array}$
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4. Solve the following subtraction problems:

a. $\begin{array}{r} 9,785.567 \\ -245.8 \\ \hline \end{array}$	b. $\begin{array}{r} 457.09 \\ -589 \\ \hline \end{array}$	c. $\begin{array}{r} 156.25 \\ -77.76 \\ \hline \end{array}$	d. $\begin{array}{r} 4100.01 \\ -3,923.21 \\ \hline \end{array}$
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e. $\begin{array}{r} 371.60 \\ -798.003 \\ \hline \end{array}$	f. $\begin{array}{r} 1,526.38 \\ -214.801 \\ \hline \end{array}$	g. $\begin{array}{r} 9,004.17 \\ -8,371.22 \\ \hline \end{array}$	h. $\begin{array}{r} 9,800.009 \\ -510 \\ \hline \end{array}$
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5. Solve the following multiplication problems:

Round the product to the nearest tenths.

a. $250 \times .032 \times 65 =$	<hr/>	<hr/>
b. $16,006 \times 70.43 =$	<hr/>	<hr/>
c. $127.003 \times 160 \times 2 =$	<hr/>	<hr/>
d. $311.71 \times 18.50 \times 30 =$	<hr/>	<hr/>
e. $\$157.75 \times 2.00 \times 98.00 =$	<hr/>	<hr/>
f. $\$169.80 \times 1,640.00 =$	<hr/>	<hr/>
g. $\$1,180 \times 35.00 =$	<hr/>	<hr/>
h. $82,000 \text{ units} \times 248 =$	<hr/>	<hr/>

6. Convert the following decimals into percentages:

- a. 0.25
- b. 9.0
- c. 0.097
- d. 2
- e. 1.76
- f. 0.55
- g. 0.75
- h. 0.2

7. Reduce the following fractions to the lowest common denominator.

a. $\frac{25}{2} = \underline{\hspace{2cm}}$

h. $\frac{77}{3} = \underline{\hspace{2cm}}$

b. $\frac{39}{24} = \underline{\hspace{2cm}}$

i. $\frac{97}{2} = \underline{\hspace{2cm}}$

c. $\frac{72}{12} = \underline{\hspace{2cm}}$

j. $\frac{50441}{41} = \underline{\hspace{2cm}}$

d. $\frac{58}{4} = \underline{\hspace{2cm}}$

k. $1378 \frac{35}{3} = \underline{\hspace{2cm}}$

e. $3 \frac{39}{39} = \underline{\hspace{2cm}}$

l. $7652 \frac{21}{7} = \underline{\hspace{2cm}}$

f. $99 \frac{4}{4} = \underline{\hspace{2cm}}$

m. $\frac{3752}{6} = \underline{\hspace{2cm}}$

g. $1079 \frac{35}{5} = \underline{\hspace{2cm}}$

n. $29 \frac{29}{29} = \underline{\hspace{2cm}}$

Practice Math Problems Answers

1. Divide $1 \frac{2}{3}$ by $1 \frac{1}{9}$:

$$1 \frac{2}{3} \div 1 \frac{1}{9}$$

Convert mix fraction to improper fraction.

$$\frac{5}{3} \quad \frac{10}{9}$$

Invert the second fraction and change to multiplication

$$\frac{5}{3} \times \frac{9}{10}$$

$$\frac{5}{3} \times \frac{9}{10} = \frac{45}{30} \quad \text{or} \quad = 1 \frac{1}{2}$$

2. Divide $12 \frac{1}{4}$ by $15 \frac{3}{4}$:

Convert mix fraction to improper fraction.

$$12 \frac{1}{4} \div 15 \frac{3}{4}$$

$$\frac{49}{4} \quad \frac{63}{4}$$

Invert the second fraction and change to multiplication

$$\frac{49}{4} \times \frac{4}{63}$$

$$\frac{49}{1} \times \frac{1}{63} = \frac{49}{63}$$

3. Solve the following addition problems:

a.	$\begin{array}{r} 326.93 \\ 25.001 \\ 1,711.9 \\ \hline 2,063.831 \end{array}$	b.	$\begin{array}{r} 25.39 \\ 17.009 \\ 0.004 \\ \hline 42.406 \end{array}$	c.	$\begin{array}{r} 67 \\ 87 \\ 4,000.00 \\ \hline 4,154.00 \end{array}$	d.	$\begin{array}{r} 190.95 \\ 305.5123 \\ 777.6 \\ \hline 1,274.0623 \end{array}$
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e.	$\begin{array}{r} \$6,000.00 \\ \$6,500.00 \\ \$7,000.00 \\ \$8,500.00 \\ \hline \$28,000.00 \end{array}$	f.	$\begin{array}{r} \$18,000.00 \\ \$36.13 \\ \$22.00 \\ \$37.48 \\ \hline \$18,095.61 \end{array}$	g.	$\begin{array}{r} 27,147.901 \\ 0.000876 \\ 16 \\ 48.1955 \\ \hline 27,212.097376 \end{array}$	h.	$\begin{array}{r} 60.108 \\ 910 \\ 1018 \\ 12 \\ \hline 2,000.108 \end{array}$
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4. Solve the following subtraction problems:

a.	$\begin{array}{r} 9,785.567 \\ -245.8 \\ \hline 9,539.767 \end{array}$	b.	$\begin{array}{r} 457.09 \\ -589 \\ \hline -131.91 \end{array}$	c.	$\begin{array}{r} 156.25 \\ -77.76 \\ \hline 78.49 \end{array}$	d.	$\begin{array}{r} 4,100.01 \\ -3,923.21 \\ \hline 176.8 \end{array}$
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e.	$\begin{array}{r} 371.60 \\ -798.003 \\ \hline -426.40 \end{array}$	f.	$\begin{array}{r} 1,526.38 \\ -214.801 \\ \hline 1,311.58 \end{array}$	g.	$\begin{array}{r} 9,004.17 \\ -8,371.22 \\ \hline 632.95 \end{array}$	h.	$\begin{array}{r} 9,800.009 \\ -510 \\ \hline 9,290.009 \end{array}$
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5. Solve the following multiplication problems:

Round the product to the nearest tenths.

a. $250 \times .032 \times 65 =$	<u>520</u>	<u>520.0</u>
b. $16,006 \times 70.43 =$	<u>1,127,302.58</u>	<u>1,127,302.6</u>
c. $127.003 \times 160 \times 2 =$	<u>40,640.960</u>	<u>40,641.0</u>
d. $311.71 \times 18.50 \times 30 =$	<u>172,999.05</u>	<u>172,999.1</u>
e. $\$157.75 \times 2.00 \times 98.00 =$	<u>\\$309.19</u>	<u>\\$309.2</u>
f. $\$169.80 \times 1,640.00 =$	<u>\\$278,472.00</u>	<u>\\$278,472.0</u>
g. $\$1,180 \times 35.00 =$	<u>\\$41,300.00</u>	<u>\\$41,300.0</u>
h. $82,000 \text{ units} \times 248 =$	<u>20,336,000</u>	<u>20,336,000.0</u>

6. Convert the following decimals into percentages:

a.	0.25	25.00%
b.	9.0	900.00%
c.	0.097	9.70%
d.	.2	200.00%
e.	1.76	176.00%
f.	0.55	55.00%
g.	0.75	75.00%
h.	0.2	20.00%

7. Reduce the following fractions to the lowest common denominator.

a.	$\frac{25}{2} =$	<u>12 1/2</u>	h.	$\frac{77}{3} =$	<u>25 2/3</u>
b.	$\frac{39}{24} =$	<u>1 15/24</u>	i.	$\frac{97}{2} =$	<u>48 1/2</u>
c.	$\frac{72}{12} =$	<u>6</u>	j.	$\frac{50,441}{41} =$	<u>1,230 11/41</u>
d.	$\frac{58}{4} =$	<u>14 1/2</u>	k.	$1,378 \frac{35}{5} =$	<u>1385</u>
e.	$3 \frac{39}{39} =$	<u>4</u>	l.	$7,652 \frac{21}{7} =$	<u>7,655</u>

f. $99 \frac{4}{4} = \underline{100}$

m. $\frac{3752}{6} = \underline{625 \frac{1}{3}}$

g. $1,079 \frac{35}{5} = \underline{1,086}$

n. $29 \frac{29}{29} = \underline{30}$